Assessment Schedule – 2006

Calculus: Integrate functions and use integrals to solve problems (90636)

Evidence and Judgement Statements

	Achievement Criteria	Q.	Evidence	Code	Judgement	Sufficiency
Achievement	Integrate functions and use integrals to solve problems.	1a	$\frac{1}{3}\sec 3x + c$	A1	Or equivalent.	**Achievement:** **Four** of Code A **including** at least **one** A1 and **one** A2.
		1b	$\frac{4}{5}e^{5x+2} + c$	A1	Or equivalent.	
		1c	$2x^{\frac{1}{2}} + 3x - 2\ln\lvert x\rvert + c$ $(2\ln\lvert kx\rvert$ or $\ln\lvert x^2\rvert + c)$ for final piece	A1	Or equivalent. Accept without | | sign. Log x not accepted.	
		2	Shaded area $= \frac{0.5}{2}[2+7+2(4+5+4+3+4)]$ $= 12.25$ units^2	A2	Units not reqd. Or equivalent. One clearly indicated copying error of values in round brackets allowed.	
		3	Cat's distance travelled: $s = \int 3 - 3\sin 3t\,dt$ $= 3t + \cos 3t + c$ Assume when $t = 0$, $s = 0$, so $s = 3t + \cos 3t - 1$ When $t = 4$, $s = 11.8$ metres Or $\int_0^4 (3 - 3\sin 3t)\,dt$ $= \left[3t + \cos 3t\right]_0^4$ $= 11.8$ m	A1 Or A2	Must show integration. Units not reqd. Or equivalent. A1 for correct indefinite integral or A2 for correct distance. One copying error of sin$3t$ allowed, but only if clearly written.	

	Achievement Criteria	Q.	Evidence	Code	Judgement	Sufficiency
Achievement with Merit	Use advanced integration techniques to find integrals and solve problems.	4	Let $u = x + 2$, then $x = u - 2$ and $\frac{\mathrm{d}x}{\mathrm{d}u} = 1$ $\int \frac{x}{\sqrt{x+2}}\,\mathrm{d}x$ $= \int \frac{u-2}{u^{\frac{1}{2}}}\,\mathrm{d}u$ $= \int u^{\frac{1}{2}} - 2u^{-\frac{1}{2}}\,\mathrm{d}u$ $= \frac{2}{3}u^{\frac{3}{2}} - 4u^{\frac{1}{2}} + c$ $= \frac{2}{3}(x+2)^{\frac{3}{2}} - 4(x+2)^{\frac{1}{2}} + c$	A1 M	(if $u = \sqrt{x+2}$, A1 for $\frac{2}{3}u^3 - 4u + c$) another form of the integral is $\frac{2}{3}(\sqrt{x+2})(x-4) + c$ If done by parts, it is $2x(x+2)^{\frac{1}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + c$ Or equivalent.	**Merit:** Achievement **plus** **Three** of Code M **or** **Four** of Code M.
		5	$\int_1^5 \frac{3x-2}{x+4}\,\mathrm{d}x$ $= \int_1^5 3 - \frac{14}{x+4}\,\mathrm{d}x$ $= \left[3x - 14\ln\lvert x+4\rvert\right]_1^5$ $= 3.77$ units^2	A1 M or A2	A1 for correct indefinite integral (if substitution of $u = x + 4$ used, A1 for $\left[3u - 14\ln\lvert u\rvert\right]$) or A2 for 3.77. Or equivalent. Units not reqd.	
		6	Volume $= \pi\int_0^{1.8} 2.25 - (y-0.3)^2\mathrm{d}y$ $= \pi\left[2.25y - \frac{(y-0.3)^3}{3}\right]_0^{1.8}$ or $\pi\left[2.16y - \frac{y^3}{3} + 0.3y^2\right]$ $= 2.916\pi$ or 9.16 m^3	A1 M or A2	A1 for correct indefinite integral or A2 for 9.16. Or equivalent. Units not reqd.	

		7	Let M = moisture content at time t $\frac{\mathrm{d}M}{\mathrm{d}t} = kM$ $\int \frac{1}{M}\,\mathrm{d}M = \int k\,\mathrm{d}t$ $\ln \mid KM \mid = kt$ $M = Ae^{kt}$ where $A = M_0$ When $t = 1$, $M = 0.5\,M_0$ $0.5\,M_0 = M_0\,\mathrm{e}^{k}$ $k = -0.69315$ (= ln0.5) [if t = 60 is used, k = –0.01155 and t = 398.6 min] $M = M_0\mathrm{e}^{-0.69315t}$ $0.01 = \mathrm{e}^{-0.69315t}$ $t = 6.64$ hours.	M or A2	Must form differential equation (otherwise best possible result A2). Accept $M = A\mathrm{e}^{kt}$ without working. Must evaluate k. Or equivalent.	

	Achievement Criteria	Q.	Evidence	Code	Judgement	Sufficiency
Achievement with Excellence	Solve more complex integration problem(s).	8	$\frac{\mathrm{d}V}{\mathrm{d}t} = -kA$ where V is volume and A is surface area. From triangles: $\frac{w}{45} = \frac{h}{30}$, $w = \frac{3h}{2}$, $A = \frac{225h}{2}$ and $V = \frac{225}{4}h^2$ $\frac{\mathrm{d}V}{\mathrm{d}h}.\frac{\mathrm{d}h}{\mathrm{d}t} = -kA$ $\frac{225h}{2}\frac{\mathrm{d}h}{\mathrm{d}t} = -k\frac{225h}{2}$ $\frac{\mathrm{d}h}{\mathrm{d}t} = -k$ $h = -kt + c$ $t = 0$, $h = 30$, $c = 30$ $h = 30 - kt$ $t = 5$, $h = 28$, $k = \frac{2}{5}$ $h = 30 - \frac{2}{5}t$ $h = 0$, $t = 75$ Water evaporates completely after 75 days.	A M E	Accept any valid method. Accept minor arithmetic error. Or equivalent.	**Excellence:** Merit **plus** Code E.
			OR $\frac{\mathrm{d}V}{\mathrm{d}t} = -kA$ Since $A = 15\sqrt{V}$, $\frac{\mathrm{d}V}{\mathrm{d}t} = -15kV$ $\int V^{-\frac{1}{2}}\,\mathrm{d}V = \int -15k\,\mathrm{d}t$ $2\sqrt{V} = -15kt + c$ When $t = 0$, $V = 50\,625$, $c = 450$ When $t = 5$, $V = 44\,100$, $k = 0.4$ $2\sqrt{V} = -6t + 450$ When $V = 0$, $t = 75$.	A1		

Judgement Statement

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Achievement	Achievement with Merit	Achievement with Excellence
Integrate functions and use integrals to solve problems. $4 \times$ A including at least $1 \times$ A1 and $1 \times$ A2	Use advanced integration techniques to find integrals and solve problems. Achievement *plus* $3 \times$ M **OR** $4 \times$ M	Solve more complex integration problems(s). Merit *plus* $1 \times$ E